Dynamic Response of Nonuniform Rotor Blades

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Theme

THE work here provides the expressions for the inertia loadings p_{x2}, p_{y2}, and p_{z2} and the inertia rotational loadings q_{x2}, q_{y2}, and q_{z2} for nonuniform rotor blades that initially are not perpendicular to the rotating axis, which is actually the case for many practical problems. In addition, the elastic axis is permitted to have abrupt discontinuities here with the inclusion of appropriate transformation equations that can facilitate the analysis of this blade configuration. Special attention is given to coupling-type terms associated with the centrifugal forces.

Contents

In Fig. 1, the x_l axis is coincident with the undeformed position of the elastic axis. The x,y,z and x_l,y_l,z_l systems of axes move with the blade around the axis of rotation at the given rotational velocity Ω. The angles λ_l and ψ_l are as indicated in the figure. The coordinates x_0,y_0,z_0 define the position 0 where an imaginary plane perpendicular to the elastic axis intersects this axis. Such a point 0 is shown in Fig. 2. This figure also shows the fixed coordinate system, X, Y, Z, the x,y,z and x_2,y_2,z_2 systems as used in the derivations, and the axes system x_3,y_3,z_3 that is parallel to the x,y,z system. Note also that the x_l,y_l,z_l and x_2,y_2,z_2 systems are parallel.

The coordinate transformation equation from the system x_2,y_2,z_2 to the system x_3,y_3,z_3 is

$$\begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} \cos\psi_l\cos\lambda_l & -\cos\psi_l\sin\lambda_l & -\sin\psi_l \\ \sin\lambda_l & \cos\lambda_l & 0 \\ \sin\psi_l\cos\lambda_l & -\sin\psi_l\sin\lambda_l & \cos\psi_l \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} \tag{1}$$

and the one from x_3, y_3, z_3 to x_2, y_2, z_2 is

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} \cos\lambda_l\cos\psi_l & \sin\lambda_l & \cos\lambda_l\sin\psi_l \\ -\sin\lambda_l\cos\psi_l & \cos\lambda_l & -\sin\lambda_l\sin\psi_l \\ -\sin\psi_l & 0 & \cos\psi_l \end{bmatrix} \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} \tag{2}$$

The inertia loadings are determined by considering the position of a fiber f in the deformed configuration of the blade and determining the displacements and accelerations in the x_2, y_2, and z_2 directions and by making use of Eq. (2). With known expressions for the accelerations a_{x2}, a_{y2}, and a_{z2} in the x_2, y_2, and z_2 directions, respectively, the inertia loadings p_{x2}, p_{y2}, and p_{z2} in the x_2, y_2, and z_2 directions, respectively, and the inertia rotational loadings q_{x2}, q_{y2}, q_{z2} about the same directions are as follows

$$\begin{aligned} p_{x_2} = & -\int_{\eta_{le}}^{\eta_{te}}\int_{-t/2}^{t/2} a_{x_2}\rho\,\mathrm{d}\zeta\,\mathrm{d}\eta \\ = & -m[\ddot{u}-\Omega^2(x_0+u\cos\lambda_l\cos\psi_l)\cos\lambda_l\cos\psi_l-\Omega^2u\sin^2\lambda_l \\ & -2\Omega\dot{v}\cos\psi_l-\Omega^2v\sin^2\psi_l\cos\lambda_l\sin\lambda_l+\Omega^2w\cos\psi_l\sin\psi_l\cos\lambda_l \\ & +\Omega^2(y_0+e_0)\sin\lambda_l-2\Omega\dot{w}\sin\psi_l\sin\lambda_l]-me[-\ddot{v}'\cos\beta \\ & -\ddot{w}'\sin\beta+\Omega^2(v'\cos\beta+w'\sin\beta)(1-\sin^2\psi_l\cos^2\lambda_l) \\ & -\Omega^2(\cos\beta+\phi\sin\beta)\sin^2\psi_l\cos\lambda_l\sin\lambda_l+\Omega^2(\sin\beta+\phi\cos\beta) \\ & \cos\psi_l\sin\psi_l\cos\lambda_l+2\Omega\dot{\phi}(\sin\beta\cos\psi_l-\cos\beta\sin\lambda_l\sin\psi_l)] \end{aligned} \tag{3}$$

$$\begin{aligned} p_{y_2} = & -\int_{\eta_{le}}^{\eta_{te}}\int_{-t/2}^{t/2} a_{y_2}\rho\,\mathrm{d}\zeta\,\mathrm{d}\eta \\ = & -m[\ddot{v}+\Omega^2x_0\cos\psi_l\sin\lambda_l-\Omega^2(y_0+e_0)\cos\lambda_l \\ & -\Omega^2u\sin^2\psi_l\cos\lambda_l\sin\lambda_l-\Omega^2v(\cos^2\psi_l\sin^2\lambda_l+\cos^2\lambda_l) \\ & -\Omega^2w\cos\psi_l\sin\psi_l\sin\lambda_l+2\Omega\dot{u}\cos\psi_l-2\Omega\dot{u}\sin\psi_l\cos\lambda_l] \\ & -me[-\ddot{\phi}\sin\beta+\Omega^2(v'\cos\beta+w'\sin\beta)\sin^2\psi_l\sin\lambda_l\cos\lambda_l \\ & -\Omega^2(\cos\beta-\phi\sin\beta)(1-\sin^2\psi_l\sin^2\lambda_l)-\Omega^2(\sin\beta+\phi\cos\beta) \\ & \cos\psi_l\sin\psi_l\sin\lambda_l-2\Omega(\dot{v}'\cos\beta+\dot{w}'\sin\beta)\cos\psi_l \\ & -2\Omega\dot{\phi}\cos\beta\sin\psi_l\cos\lambda_l] \end{aligned} \tag{4}$$

$$\begin{aligned} p_{z_2} = & -\int_{\eta_{le}}^{\eta_{te}}\int_{-t/2}^{t/2} a_{z_2}\rho\,\mathrm{d}\zeta\,\mathrm{d}\eta \\ = & -m[\ddot{w}+\Omega^2x_0\sin\psi_l+\Omega^2u\cos\psi_l\sin\psi_l\cos\lambda_l \\ & -\Omega^2v\cos\psi_l\sin\psi_l\sin\lambda_l-\Omega^2w\sin^2\psi_l+2\Omega\dot{u}\sin\lambda_l\sin\psi_l \\ & +2\Omega\dot{v}\cos\lambda_l\sin\psi_l]-me[\ddot{\phi}\cos\beta-\Omega^2(v'\cos\beta+w'\sin\beta) \\ & \cos\psi_l\sin\psi_l\cos\lambda_l-\Omega^2(\cos\beta-\phi\sin\beta)\cos\psi_l\sin\psi_l\sin\lambda_l \\ & -\Omega^2(\sin\beta+\phi\cos\beta)\sin^2\psi_l-2\Omega(\dot{v}'\cos\beta+\dot{w}'\sin\beta)\sin\psi_l\sin\lambda_l \\ & -2\Omega\dot{\phi}\sin\beta\sin\psi_l\cos\lambda_l] \end{aligned} \tag{5}$$

$$\begin{aligned} q_{x_2} = & -\int_{\eta_{le}}^{\eta_{te}}\int_{-t/2}^{t/2}[-a_{y_2}(z_2-w)+a_{z_2}(y_2-v)]\rho\,\mathrm{d}\zeta\,\mathrm{d}\eta \\ = & me[\ddot{v}\sin\beta+\ddot{v}\phi\cos\beta-\ddot{w}\cos\beta+\ddot{w}\phi\sin\beta+\Omega^2x_0\sin\beta\sin\lambda_l\cos\psi_l \\ & +\Omega^2x_0\phi\cos\beta\sin\lambda_l\cos\psi_l-\Omega^2x_0\cos\beta\sin\psi_l+\Omega^2x_0\phi\sin\beta\sin\psi_l \\ & -\Omega^2(y_0+e_0)\sin\beta\cos\lambda_l-\Omega^2(y_0+e_0)\phi\cos\beta\cos\lambda_l \\ & -\Omega^2u\sin\beta\sin^2\psi_l\cos\lambda_l\sin\lambda_l-\Omega^2u\phi\cos\beta\sin^2\psi_l\cos\lambda_l\sin\lambda_l \\ & -\Omega^2u\cos\beta\cos\psi_l\sin\psi_l\cos\lambda_l+\Omega^2u\phi\sin\beta\cos\psi_l\sin\psi_l\cos\lambda_l \\ & -\Omega^2v\sin\beta(\cos^2\psi_l\sin^2\lambda_l+\cos^2\lambda_l) \\ & -\Omega^2v\phi\cos\beta(\cos^2\psi_l\sin^2\lambda_l+\cos^2\lambda_l)+\Omega^2v\cos\beta\cos\psi_l\sin\psi_l \\ & \sin\psi_l\sin\lambda_l-\Omega^2v\phi\sin\beta\cos\psi_l\sin\psi_l\sin\lambda_l-\Omega^2w\sin\beta\sin\psi_l \\ & \cos\psi_l\sin\lambda_l-2\Omega w\phi\cos\beta\sin\psi_l\cos\psi_l\sin\lambda_l+\Omega^2w\cos\beta\sin^2\psi_l \\ & -\Omega^2w\phi\sin\beta\sin^2\psi_l+2\Omega\dot{u}\sin\beta\cos\psi_l+2\Omega\dot{u}\phi\cos\beta\cos\psi_l \\ & -2\Omega\dot{u}\cos\beta\sin\lambda_l\sin\psi_l+2\Omega\dot{u}\phi\sin\beta\sin\lambda_l\sin\psi_l-2\Omega\dot{v}\cos\beta\cos\lambda_l \\ & \sin\psi_l+2\Omega\dot{v}\phi\sin\beta\cos\lambda_l\sin\psi_l-2\Omega\dot{w}\sin\beta\sin\psi_l\cos\lambda_l \\ & -2\Omega\dot{w}\phi\cos\beta\sin\psi_l\cos\lambda_l]-\ddot{\phi}mk_m^2-\Omega^2m\cos\beta\sin\beta \\ & (k_{m2}^2-k_{m1}^2)\cos^2\psi_l+\Omega^2m\cos\beta\sin\beta(k_{m2}^2-k_{m1}^2) \\ & \sin^2\psi_l\sin^2\lambda_l+\Omega^2m\cos2\beta(k_{m2}^2-k_{m1}^2)\cos\psi_l\sin\psi_l\sin\lambda_l \end{aligned}$$

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Index categories: Propeller and Rotor Systems; Vibration; Aerodynamics.

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\begin{aligned}
&-\Omega^2\phi m(k_{m2}^2-k_{m1}^2)\cos2\beta+\Omega^2\phi m\cos2\beta\,(k_{m2}^2-k_{m1}^2)\sin^2\psi_l\sin^2\lambda_l+\Omega^2\phi m\cos2\beta(k_{m2}^2-k_{m1}^2)\sin^2\psi_l-4\Omega^2\phi m\cos\beta\sin\beta\\
&(k_{m2}^2-k_{m1}^2)\cos\psi_l\sin\psi_l\sin\lambda_l+\Omega^2\phi^2m(k_{m2}^2-k_{m1}^2)\cos\beta\sin\beta-\Omega^2\phi^2m\cos\beta\sin\beta(k_{m2}^2-k_{m1}^2)\sin^2\psi_l\sin^2\lambda_l-\Omega^2\phi^2m\cos\beta\sin\beta\\
&(k_{m2}^2-k_{m1}^2)\sin^2\psi_l-\Omega^2\phi^2m\cos2\beta(k_{m2}^2-k_{m1}^2)\cos\psi_l\sin\psi_l\sin\lambda_l+\Omega^2v'm\cos\beta\sin\beta(k_{m2}^2-k_{m1}^2)\sin^2\psi_l\sin\lambda_l\cos\lambda_l+\Omega^2v'm\\
&(k_{m2}^2\cos^2\beta+k_{m1}^2\sin^2\beta)\cos\psi_l\sin\psi_l\cos\lambda_l-\Omega^2v'\phi m\cos\beta\sin\beta(k_{m2}^2-k_{m1}^2)\cos\psi_l\sin\psi_l\cos\lambda_l+\Omega^2v'\phi m(k_{m2}^2\cos^2\beta+k_{m1}^2\sin^2\beta)\\
&\sin^2\psi_l\sin\lambda_l\cos\lambda_l+\Omega^2w'm\cos\beta\sin\beta(k_{m2}^2-k_{m1}^2)\cos\psi_l\sin\psi_l\cos\lambda_l+\Omega^2w'm(k_{m2}^2\sin^2\beta+k_{m1}^2\cos^2\beta)\sin^2\psi_l\sin\lambda_l\cos\lambda_l\\
&+\Omega^2w'\phi m\cos\beta\sin\beta(k_{m2}^2-k_{m1}^2)\sin^2\psi_l\sin\lambda_l\cos\lambda_l-\Omega^2w'\phi m(k_{m2}^2\sin^2\beta+k_{m1}^2\cos^2\beta)\cos\psi_l\sin\psi_l\cos\lambda_l-2\Omega\dot v'm\cos\beta\sin\beta\\
&(k_{m2}^2-k_{m1}^2)\cos\psi_l+2\Omega\dot v'm(k_{m2}^2\cos^2\beta+k_{m1}^2\sin^2\beta)\sin\psi_l\sin\lambda_l-2\Omega\dot v'\phi m\cos\beta\sin\beta(k_{m2}^2-k_{m1}^2)\sin\psi_l\sin\lambda_l-2\Omega\dot v'\phi m(k_{m2}^2\cos^2\beta\\
&+k_{m1}^2\sin^2\beta)\cos\psi_l+2\Omega\dot w'm\cos\beta\sin\beta(k_{m2}^2-k_{m1}^2)\sin\psi_l\sin\lambda_l-2\Omega\dot w'm(k_{m2}^2\sin^2\beta+k_{m1}^2\cos^2\beta)\cos\psi_l-2\Omega\dot w'\phi m\cos\beta\sin\beta\\
&(k_{m2}^2-k_{m1}^2)\cos\psi_l-2\Omega\dot w'\phi m(k_{m2}^2\sin^2\beta+k_{m1}^2\cos^2\beta)\sin\psi_l\sin\lambda_l-2\Omega\dot\phi\phi mk_m^2\sin\psi_l\cos\lambda_l
\end{aligned}
\tag{6}
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\begin{aligned}
q_{y_2}=&-\int_{\eta_{te}}^{\eta_{le}}\int_{-t/2}^{t/2}-a_{x_2}(z_2-w)\rho\,\mathrm{d}\zeta\mathrm{d}\eta\\
=&\,me[\ddot u\sin\beta-2\Omega\dot v\sin\beta\cos\psi_l-2\Omega\dot v\phi\cos\beta\cos\psi_l-2\Omega\dot w\sin\beta\sin\psi_l\sin\lambda_l-2\Omega\dot w\phi\cos\beta\sin\psi_l\sin\lambda_l+\ddot u\phi\cos\beta-\Omega^2x_0\sin\beta\cos\lambda_l\cos\psi_l\\
&-\Omega^2\sin\beta(y_0+e_0)\sin\lambda_l-\Omega^2x_0\phi\cos\beta\cos\lambda_l\cos\psi_l-\Omega^2\phi\cos\beta(y_0+e_0)\sin\lambda_l-\Omega^2u\sin\beta\cos^2\lambda_l\cos^2\psi_l-\Omega^2u\sin\beta\sin^2\lambda_l\\
&-\Omega^2u\phi\cos\beta(\sin^2\lambda_l+\cos^2\lambda_l\cos^2\psi_l)-\Omega^2v\sin\beta\sin^2\psi_l\cos\lambda_l\sin\lambda_l-\Omega^2v\phi\cos\beta\sin^2\psi_l\cos\lambda_l\sin\lambda_l+\Omega^2w\sin\beta\cos\psi_l\sin\psi_l\cos\lambda_l\\
&+\Omega^2w\phi\cos\beta\cos\psi_l\sin\psi_l\cos\lambda_l]-m\ddot v'(k_{m2}^2-k_{m1}^2)\cos\beta\sin\beta-m\ddot w'(k_{m2}^2\sin^2\beta+k_{m1}^2\cos^2\beta)-m\ddot v'\phi(k_{m2}^2\cos^2\beta+k_{m1}^2\sin^2\beta)\\
&-m\ddot w'\phi(k_{m2}^2-k_{m1}^2)\cos\beta\sin\beta-\Omega^2m\cos\beta\sin\beta(k_{m2}^2-k_{m1}^2)\sin^2\psi_l\cos\lambda_l\sin\lambda_l+\Omega^2m(k_{m2}^2\sin\beta+k_{m1}^2\cos\beta)\cos\psi_l\sin\psi_l\cos\lambda_l\\
&-\Omega^2m\phi\cos2\beta(k_{m2}^2-k_{m1}^2)\sin^2\psi_l\cos\lambda_l\sin\lambda_l+2\Omega^2m\phi\cos\beta\sin\beta(k_{m2}^2-k_{m1}^2)\cos\psi_l\sin\psi_l\cos\lambda_l+\Omega^2\phi^2m\cos\beta\sin\beta\\
&(k_{m2}^2-k_{m1}^2)\sin^2\psi_l\cos\lambda_l\sin\lambda_l+\Omega^2\phi^2m(k_{m2}^2\cos^2\beta-k_{m1}^2\sin^2\beta)\cos\psi_l\sin\psi_l\cos\lambda_l+\Omega^2v'm\cos\beta\sin\beta(k_{m2}^2-k_{m1}^2)\\
&(1-\sin^2\psi_l\cos^2\lambda_l)+\Omega^2v'\phi(k_{m2}^2\cos^2\beta+k_{m1}^2\sin^2\beta)(1-\sin^2\psi_l\cos^2\lambda_l)+\Omega^2w'm(k_{m2}^2\sin^2\beta+k_{m1}^2\cos^2\beta)(1-\sin^2\psi_l\cos^2\lambda_l)\\
&+\Omega^2w'\phi m\cos\beta\sin\beta(k_{m2}^2-k_{m1}^2)(1-\sin^2\psi_l\cos^2\lambda_l)+2\Omega\dot\phi m(k_{m2}^2\sin^2\beta+k_{m1}^2\cos^2\beta)\cos\psi_l-2\Omega\dot\phi m\cos\beta\sin\beta(k_{m2}^2-k_{m1}^2)\sin\psi_l\\
&\sin\lambda_l+2\Omega\dot\phi\phi m\cos\beta\sin\beta(k_{m2}^2-k_{m1}^2)\cos\psi_l-2\Omega\dot\phi\phi m(k_{m2}^2\cos^2\beta+k_{m1}^2\sin^2\beta)\sin\psi_l\sin\lambda_l
\end{aligned}
\tag{7}
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\begin{aligned}
q_{z_2}=&-\int_{\eta_{te}}^{\eta_{le}}\int_{-t/2}^{t/2}[-a_{x_2}(y_2-v)]\rho\,\mathrm{d}\zeta\mathrm{d}\eta\\
=&\,me[\ddot u\cos\beta-2\Omega\dot v\cos\beta\cos\psi_l-2\Omega\dot w\cos\beta\sin\psi_l\sin\lambda_l+2\Omega\dot v\phi\sin\beta\cos\psi_l+2\Omega\dot w\phi\sin\beta\sin\psi_l\sin\lambda_l-\Omega^2x_0\cos\beta\cos\psi_l\cos\lambda_l+\Omega^2x_0\phi\sin\beta\\
&\cos\psi_l\cos\lambda_l-\Omega^2\cos\beta(y_0+e_0)\sin\lambda_l+\Omega^2\phi\sin\beta(y_0+e_0)\sin\lambda_l-\Omega^2u\cos\beta(1-\cos^2\lambda_l\sin^2\psi_l)+\Omega^2u\phi\sin\beta(1-\cos^2\lambda_l\sin^2\psi_l)\\
&-\Omega^2v\cos\beta\sin^2\psi_l\cos\lambda_l\sin\lambda_l+\Omega^2v\phi\sin\beta\sin^2\lambda_l\cos\lambda_l\sin\lambda_l+\Omega^2w\cos\beta\cos\psi_l\sin\psi_l\cos\lambda_l-\Omega^2w\phi\sin\beta\cos\psi_l\sin\psi_l\cos\lambda_l-\ddot u\phi\sin\beta]\\
&-\ddot w'm(k_{m2}^2-k_{m1}^2)\cos\beta\sin\beta-\ddot v'm(k_{m2}^2\cos^2\beta+k_{m1}^2\sin^2\beta)+\ddot w'\phi m(k_{m2}^2\sin^2\beta+k_{m1}^2\cos^2\beta)+\ddot v'\phi m(k_{m2}^2-k_{m1}^2)\cos\beta\sin\beta\\
&-\Omega^2m(k_{m2}^2\cos^2\beta+k_{m1}^2\sin^2\beta)\sin^2\psi_l\cos\lambda_l\sin\lambda_l+\Omega^2m\cos\beta\sin\beta(k_{m2}^2-k_{m1}^2)\cos\psi_l\sin\psi_l\cos\lambda_l+2\Omega^2\phi m\cos\beta\sin\beta(k_{m2}^2-k_{m1}^2)\\
&\sin^2\psi_l\cos\lambda_l\sin\lambda_l+\Omega^2\phi m\cos2\beta(k_{m2}^2-k_{m1}^2)\cos\psi_l\sin\psi_l\cos\lambda_l-\Omega^2\phi^2m(k_{m2}^2\sin^2\beta+k_{m1}^2\cos^2\beta)\sin^2\psi_l\cos\lambda_l\sin\lambda_l\\
&-\Omega^2\phi^2m\cos\beta\sin\beta(k_{m2}^2-k_{m1}^2)\cos\psi_l\sin\psi_l\cos\lambda_l+\Omega^2v'm(k_{m2}^2\cos^2\beta+k_{m1}^2\sin^2\beta)(1-\sin^2\psi_l\cos^2\lambda_l)-\Omega^2v'\phi m\cos\beta\sin\beta\\
&(k_{m2}^2-k_{m1}^2)(1-\sin^2\psi_l\cos^2\lambda_l)+\Omega^2w'm\cos\beta\sin\beta(k_{m2}^2-k_{m1}^2)(1-\sin^2\psi_l\cos^2\lambda_l)-\Omega^2w'\phi m(k_{m2}^2\sin^2\beta+k_{m1}^2\cos^2\beta)\\
&(1-\sin^2\psi_l\cos^2\lambda_l)+2\Omega\dot\phi m\cos\beta\sin\beta(k_{m2}^2-k_{m1}^2)\cos\psi_l-2\Omega\dot\phi m(k_{m2}^2\cos^2\beta+k_{m1}^2\sin^2\beta)\sin\psi_l\sin\lambda_l+2\Omega\dot\phi\phi m\cos\beta\sin\beta\\
&(k_{m2}^2-k_{m1}^2)\sin\psi_l\sin\lambda_l-2\Omega\dot\phi\phi m(k_{m2}^2\sin^2\beta+k_{m1}^2\cos^2\beta)\cos\psi_l
\end{aligned}
\tag{8}
$$

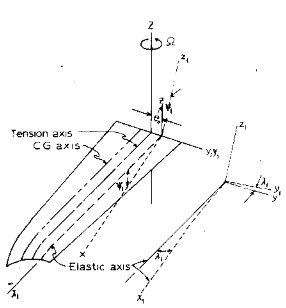

Fig. 1 Rotational configuration of a nonuniform rotor blade.

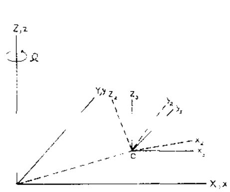

Fig. 2 Fixed and moving coordinate systems of axes as used in transformation equations and in the derivations.

The details of the derivations are shown in the full paper. In the above equations, u, v, and w are the displacements in the x_l, y_l, and z_l axes, respectively, m is the mass of blade per unit length, β is the blade angle prior to any deformation, η and ζ are coordinates along the major axis and perpendicular to it, η_{te} and η_{le} are values of η for trailing edge and leading edge of cross section, ρ is mass density, ϕ is the angle of twist, k_m is the polar radius of gyration of the cross-sectional mass about the elastic axes, and k_{m1}, k_{m2} are mass radii of gyration about major neutral axis and about an axis perpendicular to chord through the elastic axis.

If the axial deformation u and the higher order terms are small, the above equations can be simplified by neglecting these terms. If also $\lambda_l=\psi_l=y_0=0$, the above equations reduce to those by Houbolt and Brooks in Ref. 2. The equations of motion for the rotor blade, which include the above expressions for the inertia loadings, as well as additional information, are included in the full paper.

References

[1]Fertis, D. G., *Dynamics and Vibration of Structures*, John Wiley and Sons, New York, 1973.

[2]Houbolt, J. D. and Brooks, G. W., "Differential Equations of Motion for Combined Flapwise Bending, Chordwise Bending, and Torsion of Twisted Nonuniform Rotor Blades," Langley Aeronautical Laboratory, Langley Field, Va., Rept. 1346, 1956.